## Population Growth

I. Geometric growth
II. Exponential growth
III. Logistic growth

## Bottom line:

When there are no limits, populations grow faster,
and FASTER and FASTER!


## The Simple Case: Geometric Growth

- Constant reproduction rate
- Non-overlapping generations (like annual plants, insects)
- Also, discrete breeding seasons (like birds, trees, bears)
- Suppose the initial population size is 1 individual.
- The indiv. reproduces once \& dies, leaving 2 offspring.
- How many if this continues?



## Equations for Geometric Growth

- Growth from one season to the next: $\mathrm{N}_{\mathrm{t}+1}=\mathrm{N}_{\mathrm{t}} \lambda$, where:
- $\mathrm{N}_{\mathrm{t}}$ is the number of individuals at time t
- $\mathrm{N}_{\mathrm{t}+1}$ is the number of individuals at time $\mathrm{t}+1$
- $\lambda$ is the rate of geometric growth
- If $\lambda>1$, the population will increase
- If $\lambda<1$, the population will decrease
- If $\lambda=1$, the population will stay unchanged


## Equations for Geometric Growth

- From our previous example, $\boldsymbol{\lambda}=2$
- If $\mathrm{N}_{\mathrm{t}}=4$, how many the next breeding cycle?
- $\mathrm{N}_{\mathrm{t}+1}=\mathrm{N}_{\mathrm{t}} \lambda=(4)(2)=8$
$\cdot$ How many the following breeding cycle?
- $\mathrm{N}_{\mathrm{t}+1}=(4)(2)(2)=16$
- In general, with knowledge of the initial N and $\lambda$, one can estimate N at any time in the future by:
- $\mathrm{N}_{\mathrm{t}}=\mathrm{N}_{0} \lambda^{\mathrm{t}}$


## Using the Equations for Geometric Growth

- If $\mathrm{N}_{0}=2, \lambda=2$, how many after 5 breeding cycles?
- $\mathrm{N}_{\mathrm{t}}=\mathrm{N}_{0} \lambda^{\mathrm{t}}=(2)(2)^{5}=(2)(32)=64$
- If $\mathrm{N}_{0}=1000, \lambda=2$, how many after 5 breeding cycles?
- $\mathrm{N}_{\mathrm{t}}=\mathrm{N}_{0} \lambda^{\mathrm{t}}=(1000)(2)^{5}=(1000)(32)=32,000$


## Using the Equations for Geometric Growth

- If in 2001, there were 500 black bears in the Pasayten Wilderness, and there were 600 in 2002, how many would there be in 2010 ?
- First, estimate $\lambda$ :
- If $\mathrm{N}_{\mathrm{t}+1}=\mathrm{N}_{\mathrm{t}} \lambda$, then $\lambda=\mathrm{N}_{\mathrm{t}+1} / \mathrm{N}_{\mathrm{t}}$, or $600 / 500=1.2$
-If $\mathrm{N}_{0}=500, \lambda=1.2$, then in 2010 ( 9 breeding cycles later)
- $\mathrm{N}_{9}=\mathrm{N}_{0} \lambda^{9}=(500)(1.2)^{9}=2579$
- In 2060 (59 breeding seasons), $\mathrm{N}=23,478,130$ bears!



## Exponential Growth - Continuous Breeding

r explained:
$\mathrm{r}=\mathrm{b}-\mathrm{d}$, where b is the birth rate, and d is the death rate
Both are expressed in units of indivs/indiv/unit time
When $b>d, r>0$, and $d N / d t(=r N)$ is positive
When $b<d, r<0$, and $d N / d t$ is negative
When $\mathrm{b}=\mathrm{d}, \mathrm{r}=0$, and $\mathrm{dN} / \mathrm{dt}=0$

## Exponential Growth - Continuous Breeding

$\mathrm{dN} / \mathrm{dt}=\mathrm{rN}$, where
$\mathrm{dN} / \mathrm{dt}$ is the instantaneous
rate of change
$r$ is the intrinsic rate of increase

## Equations for Exponential Growth

- If $\mathrm{N}=100$, and $\mathrm{r}=0.1$ indivs/indiv/day, how much growth in one day?
- $\mathrm{dN} / \mathrm{dt}=\mathrm{rN}=(0.1)(100)=10$ individuals
- To predict N at any time in the future, one needs to solve the differential equation:
- $\mathrm{N}_{\mathrm{t}}=\mathrm{N}_{0} \mathrm{e}^{\mathrm{rt}}$

Exponential Growth in Rats
•In Norway rats that invade a new warehouse with
ideal conditions, $\mathrm{r}=0.0147$ indivs/indiv/day
•If $\mathrm{N}_{0}=10$ rats, how many at the end of 100 days?
• $\mathrm{N}_{\mathrm{t}}=\mathrm{N}_{0} \mathrm{e}^{\mathrm{rt}}$, so $\mathrm{N}_{100}=10 \mathrm{e}^{(0.0147)(100)}=43.5$ rats
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## Comparing Exponential and Geometric Equations:

- Geometric: $\mathrm{N}_{\mathrm{t}}=\mathrm{N}_{0} \lambda^{\mathrm{t}}$
- Exponential: $\mathrm{N}_{\mathrm{t}}=\mathrm{N}_{0} \mathrm{e}^{\mathrm{rt}}$
- Thus, a reasonable way to compare growth
parameters is: $\mathrm{e}^{\mathrm{r}}=\lambda$, or $\mathrm{r}=\ln (\lambda)$


## Assumptions of the Equations

- All individuals reproduce equally well.
- All individuals survive equally well.
- Conditions do not change through time.


## I. Points about exponential growth



What happens if there ARE limits?
(And eventually there ALWAYS are!)
LOGISTIC POPULATION GROWTH

