Population Growth

I. Geometric growth II. Exponential growth III. Logistic growth

Bottom line:

When there are no limits, populations grow faster, and FASTER and FASTER!











- Constant reproduction rate
- Non-overlapping generations (like annual plants, insects)
- Also, discrete breeding seasons (like birds, trees, bears)
- Suppose the initial population size is 1 individual.
- The indiv. reproduces once & dies, leaving 2 offspring.
- How many if this continues?



Equations for Geometric Growth

- Growth from one season to the next: $N_{t+1} = N_t \lambda$, where:
 - N_t is the number of individuals at time t
 - N_{t+1} is the number of individuals at time t+1
 - $\bullet \, \lambda$ is the rate of geometric growth
 - If $\lambda > 1$, the population will increase
 - If $\lambda < 1$, the population will decrease
 - If $\lambda = 1$, the population will stay unchanged

Equations for Geometric Growth

From our previous example, λ = 2
If N_t = 4, how many the next breeding cycle?

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$$N_{t+1} = N_t \lambda = (4)(2) = 8$$

- •How many the following breeding cycle? • $N_{t+1} = (4)(2)(2) = 16$
- In general, with knowledge of the initial N and λ, one can estimate N at any time in the future by:
 N_t = N₀ λ^t

Using the Equations for Geometric Growth

- If N₀ = 2, λ = 2, how many after 5 breeding cycles?
 N_t = N₀ λ^t = (2)(2)⁵ = (2)(32) = 64
 If N₀ = 1000, λ = 2, how many after 5 breeding cycles?
- $N_0 = 1000, \lambda = 2$, now many after 5 breeding cycles? • $N_t = N_0 \lambda^t = (1000)(2)^5 = (1000)(32) = 32,000$

What does "faster" mean?

Growth rate vs. number of new individuals

Using the Equations for Geometric Growth

• If in 2001, there were 500 black bears in the Pasayten Wilderness, and there were 600 in 2002, how many would there be in 2010?

- First, estimate λ : • If $N_{t+1} = N_t \lambda$, then $\lambda = N_{t+1}/N_t$, or 600/500 = 1.2
- •If $N_0 = 500$, $\lambda = 1.2$, then in 2010 (9 breeding cycles later) • $N_9 = N_0 \lambda^9 = (500)(1.2)^9 = 2579$

• In 2060 (59 breeding seasons), N = 23,478,130 bears!



Exponential Growth - Continuous Breeding

dN/dt = rN, where

dN/dt is the instantaneous rate of change

r is the intrinsic rate of increase

Exponential Growth - Continuous Breeding

r explained:

r = b - d, where b is the birth rate, and d is the death rate

Both are expressed in units of indivs/indiv/unit time

When b>d, r>0, and dN/dt (=rN) is positive When b<d, r<0, and dN/dt is negative When b=d, r=0, and dN/dt = 0

Equations for Exponential Growth

- If N = 100, and r = 0.1 indivs/indiv/day, how much growth in one day?
 - dN/dt = rN = (0.1)(100) = 10 individuals
- To predict N at any time in the future, one needs to solve the differential equation:
 N_t = N₀e^{rt}

Exponential Growth in Rats

• In Norway rats that invade a new warehouse with ideal conditions, r = 0.0147 indivs/indiv/day

- If $N_0 = 10$ rats, how many at the end of 100 days?
- $N_t = N_0 e^{rt}$, so $N_{100} = 10e^{(0.0147)(100)} = 43.5$ rats

Comparing Exponential and Geometric Equations:

- Geometric: $N_t = N_0 \lambda^t$
- Exponential: $N_t = N_0 e^{rt}$
- Thus, a reasonable way to compare growth parameters is: $e^r = \lambda$, or $r = ln(\lambda)$

Assumptions of the Equations

- All individuals reproduce equally well.
- All individuals survive equally well.
- Conditions do not change through time.

I. Points about exponential growth





What happens if there ARE limits? (And eventually there ALWAYS are!)

LOGISTIC POPULATION GROWTH